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Heat kernel estimates for the fractional Laplacian

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I will report a joint work with **Tomasz Grzywny** and **Michał Ryznar** from Wrocław University of Technology, Poland (the paper is on arXiv):

Explicit sharp estimates for the Green function of the Laplacian in $C^{1,1}$ domains were given in 1986 by Zhao. Sharp estimates of the Green function of Lipschitz domains were given in 2000 by Bogdan. Explicit qualitatively sharp estimates for the classical heat kernel in $C^{1,1}$ domains were established in 2002 by Zhang. Qualitatively sharp heat kernel estimates in Lipschitz domain were given in 2003 by Varopulous.

The development of the boundary potential theory of the fractional Laplacian follows a parallel path. Green function estimates were obtained in 1997 and 1998 by Kulczycki and Chen and Song for $C^{1,1}$ domains, and in 2002 by Jakubowski for Lipschitz domains. In 2008 Chen, Kim and Song gave a sharp and explicit estimate for the heat kernel $p_D(t,x,y)$ of the fractional Laplacian on $C^{1,1}$ domains D. Our contribution is the following approximate factorization.

Theorem *If D is a Lipschitz domain then for $0 < t \leq 1$ and all $x, y \in \mathbf{R}^d$,*

$$C^{-1}P^x(\tau_D > t)P^y(\tau_D > t) \leq \frac{p_D(t,x,y)}{p(t,x,y)} \leq CP^x(\tau_D > t)P^y(\tau_D > t)\,. \qquad (1)$$

Here $p(t,x,y)$ is the heat kernel of the fractional Laplacian on $\mathbf{R}^d$, and

$$P_x(\tau_D > t) = \int_{\mathbf{R}^d} p_t^D(x,y)dy$$

is the *survival probability* of the corresponding isotropic α-stable Lévy process.

For some unbounded sets D the time range for (1) can be $0 < t < \infty$.

We complement the result by estimates of the survival probability.